Mushroom picking heuristics framework for knapsack-like problems of resource allocation

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***Abstract*—Resource allocation is a complex challenge that extends across diverse disciplines, each presenting its distinct considerations and demands. This intricate task involves the distribution of resources in a manner that meets the needs and objectives of various sectors. In this study, we propose an innovative mushroom picker heuristics to knapsack-like resource allocation problems, mainly with product categorization, wherein each potential solution is metaphorically likened to a mushroom. The heuristic process comprises several stages: first, the preparation of the forest ground, followed by the identification of distinct mushroom clearings, then the search for mushrooms within these clearings, and finally, the decision-making process regarding the selection and collection of mushrooms. Through this heuristic framework, we aim to elucidate effective strategies for solution discovery and decision-making in complex problem domains. Twelve tuning parameters are presented to reduce the solution space. We provide an explanation of the application of the proposed mushroom picking heuristics on the basis of two problems: (1) the shelf space allocation in retail and (2) the commercial to TV break placement in media planning. This algorithm can also be used to solve other problems that can be modelled as knapsack problems.**

Keywords— resource allocation problem, shelf space allocation problem, media planning, knapsack, heuristics.

I. INTRODUCTION

Resource allocation presents an overall challenge spanning diverse domains, including management, economics, engineering, and operations research, where it answers the purpose of the fundamental determinant of efficiency and achievement.

In the areas of management and economics, resources constitute a broad spectrum of assets, encompassing savings, investments, and expenditures, collectively shaping the overall cost structure of production or service delivery. In the realm of manufacturing, resources manifest as indispensable elements of the production continuum, comprising materials, equipment, labour, and financial capital, each exerting a significant influence on the quality and outcome of goods or services.

From economics to environmental science, from healthcare to education, the allocation of resources demands a nuanced understanding of the intricate interplay between supply, demand, efficiency, and equity. This multifaceted nature underscores the necessity for interdisciplinary collaboration and innovative approaches to effectively address the complexities inherent in resource allocation.

In project management, where the primary objective revolves around delivering projects or programs within specified parameters, resources are embodied by personnel, software, or equipment. Effective allocation of these resources is pivotal for ensuring project milestones are met and objectives are achieved.

In the realm of telecommunication networks, resources are typified by bandwidth allocation, representing the finite capacity of channels shared among users. Efficient management of bandwidth resources is crucial for maintaining network performance and user satisfaction.

In the field of medicine, resources span a spectrum of medical and health-related provisions, including materials, personnel, and financial assets, all of which must be judiciously distributed among diverse patient populations. Optimal resource allocation in healthcare is indispensable for ensuring equitable access to quality care and maximizing health outcomes.

Hence, the task of resource allocation extends throughout

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multiple fields, necessitating sophisticated methodologies to maximize resource efficiency and achieve specific goals. By employing strategic allocation strategies and effective management techniques, organizations can successfully navigate intricate landscapes, boost productivity, and promote sustainable development.

Knapsack problems involve optimizing the allocation of limited resources to maximize utility while considering constraints. In our context, the inclusion of product categorization adds complexity, as items may belong to different categories with varying characteristics and priorities. The term “mushroom picker heuristics” refers to a novel approach inspired by human activity aimed at searching and collecting mushrooms.

Mushroom picker efficiently navigates the forest and collects mushrooms (resources) while considering factors such as resource quality and proximity. In our study, we introduce mushroom picker heuristics for resource optimization, incorporating two distinct sorting rule variants. These heuristic strategies are formulated to streamline the process of finding solutions, thereby improving efficiency and fostering profitability. We give an example of how the proposed heuristics could be applied to the retail shelf space allocation problem (SSAP) and media planning advertisement allocation problem (AAP). We deployed a collection of 12 and 13 carefully chosen parameters for SSAP and AAP, respectively, with the strategic goal of reducing the scope of potential solutions. Through this approach, our objective is to develop solutions that are comparatively gainful while avoiding dependence on random factors and bypassing exhaustive exploration of the entire solution space.

The remainder of the paper is organized as follows. Section 2 introduces the related literature concerning two investigated problems of resource allocation. Section 3 presents the knapsack, shelf space allocation and media planning problems. In Section 4, we describe the new mushroom picker heuristics. Section 5 presents the usage of the novel heuristics in solving the shelf space allocation and media planning problems. Conclusions are given in Section 6.

II. Related literature

A. *Heuristics in shelf space allocation*

In the retail industry, effective management of product selection and shelf space allocation is crucial for optimizing both customer satisfaction and store profitability. Retailers are faced with the challenge of not only selecting the right mix of products to meet diverse customer preferences but also arranging these products in a manner that maximizes visibility and appeal. This dual task involves strategic decisions and careful planning to ensure that the limited shelf space is used to its fullest potential, accommodating an optimal assortment of products that can drive sales and enhance the shopping experience.

The retailer’s primary objective is to maximize the profit garnered from these sales. This goal presents a complex task in retail management, in which the challenge encompasses the decision-making process regarding the determination of appropriate allocation of shelf space for each product of the selected assortment and the strategic placement of these products on the shelves to maximize sales or profits.

In its most basic form, the product selection problem is deciding which products to display in the limited amount of shelf space that is available, whereas the shelf space allocation problem is determining how the chosen products will actually be arranged on shelves and how many units of each product will be displayed (Landa-Silva, 2009).

Both the product selection and shelf space allocation problems are interrelated and crucial for retail success. Once the products have been chosen, the focus shifts to their optimal arrangement on the shelves. This arrangement not only needs to be visually appealing but also strategically planned to enhance product visibility and accessibility.

The appropriate product placement on shelves helps retailers in two ways: it lowers the cost of inventory and shelf replenishment while also boosting sales. Numerous aspects, including the product’s placement on the shelf, its facings, and its surrounding items, affect how many units a product sells (Drèze et al., 1994).

The profitability of every retail establishment, regardless of size, is significantly influenced by the way shelf space is managed (Yang & Chen, 1999; Lim et al.,2004; Czerniachowska et al., 2021)). The heuristics for practical shelf space allocation problems, particularly with visible horizontal and vertical grouping of products, were developed in some research (Czerniachowska et al., 2022; Czerniachowska&, Hernes, 2020; Czerniachowska et al.,2021). The techniques suggested in the studies can help retailers streamline their category management decisions, making the process more efficient. By leveraging the proposed heuristics, retailers can react more quickly to market changes and consumer demands, leading to better-informed decisions.

Effective shelf space allocation directly influences inventory management and sales performance, making it a vital component of retail strategy. Maximizing profits, cutting expenses, and raising customer happiness all depend on the effective use and careful management of shelf space, a scarce but essential resource (Landa-Silva, 2009).

Factors such as product demand, popularity, shelf-life, physical dimensions, and profitability are critical considerations in addressing this issue. The overarching objective is to effectively utilize the available shelf space to drive increased sales, mitigate stockouts, and bolster the overall performance of the retail establishment.

Researchers have suggested using heuristic modelling to obtain near-optimal solutions to the shelf-space design because retail shelf-space problems remain complex in nature, and managers lack error-free estimates of the parameters that affect product performance (Borin & Farris, 1995) . One such researcher is Yang (2001). Heuristics are algorithmic approaches that are intended to find a near-optimal solution (profit, sales, product movement in this case) as fast as possible, even though they might not yield the best result. Yang proposed a three-phase heuristic to deal with the shelf space allocation problem, which was applicable to large retails stores (Yang, 2001). The method consisted of the following steps: the preparatory phase, allocation phase, adjustment phase, and termination phase. Landa-Silva et al. (2009) improved Yang's (2001) heuristics and proposed its adaptation to real large stores. The suggested heuristics made an initial arrangement and then iteratively refined the obtained solution by applying adjustment movements. The suggested by Hansen et al. (2010) heuristic approach included two main phases and was a variation of Yang (2001) (Yang, 2001; Hansen, 2010). Initially, the algorithm ranked all products based just on average profit, disregarding shelf placement.

According to Reeves (1996), a heuristic is a method that finds near-optimal solutions at a reasonable computing cost without providing any guarantees of optimality or practicality. Heuristics include things like hill-climbing tactics and greedy algorithms. They have the drawback of perhaps becoming trapped in a local optimum despite their simplicity(Reeves, 1996).

Unlike exact algorithms that guarantee finding the best possible solution, heuristics aim for good-enough solutions, trading off some degree of accuracy for efficiency and speed. These methods are particularly useful in complex or large-scale problems where traditional methods may be too slow or impractical.

Heuristic solutions were constructed based on the models proposed by several authors (Borin& Farris,1995; Yang, 2001; Borin et al., 1994; Urban;1998). In an inventory-theoretic approach to shelf-space allocation, Urban (1998) employed a greedy heuristic and a genetic algorithm for the solution of an integrated problem, while Yang used adjustment heuristics. Borin et al. (1994) used a heuristic technique based on simulated annealing.

Most of the heuristics are based on straightforward, intuitive principles that are easily used in real-world scenarios to carry out choices on how to allocate shelf space. In order to maximize profitability under various limitations, such as limited shelf space and elasticity factors, Binguler et al. (2015) provided a novel heuristic to find an optimal allocation of shelf space for various items (Binguler, 2015).

Gajjar and Adil (2011a) examined a retail shelf-space allocation issue in which the retailer aimed to distribute the available spaces across several shelves among numerous items while taking into account the direct space elasticity of the product's demand. To address this issue, Gajjar and Adil (2011a) suggested using the dynamic programming heuristic to find a close-to-optimal solution in an acceptable amount of time (Gajjar & Adil, 2011a). In order to create an effective heuristic, Gajjar and Adil (2011b) suggested a novel initial constructor and a neighbourhood move approach. The heuristics suggested are competitive with those that are already in use. Gajjar and Adil (2011b) examined a retail shelf space allocation problem with a linear profit function and created quick and effective heuristics to solve it using a neighbourhood search technique and a new initial constructor (Gajjar & Adil, 2011b).

A heuristic solution strategy was used in Borin et al.'s (1994) study, which examined product assortment and space allocation in a limited optimization problem. The study came to the conclusion that disregarding the impacts of out-of-stock and product assortment results in suboptimality. In a recent empirical study on the responsiveness of product category sales to shelf space allocation, Desmet and Renaudin (1998) employed the Corstjens and Dolye (1983) model framework. This model is based on a demand function that links the share of sales to the share of space allotted to the product category. The findings indicate that space elasticities rise in proportion to the product category's rate of impulse purchases(Desmet& Renaudin, 1998; Corstjens & Doyle, 1983).

Three contributions to the retail shelf space allocation problem taking space elasticity into consideration (SSAPSE) are presented in the study by Gajjar and Adil (2010). First, Gajjar and Adil (2010) used piecewise linearization to transform an existing non-linear model for SSAPSE into an integer programming (IP) model. Secondly, Gajjar and Adil (2010) demonstrated that the suggested IP model's linear programming relaxation yielded a tight upper bound. Third, Gajjar and Adil (2010) created a heuristic that reliably yielded close to optimal solutions for problems of a given size that are generated at random (Gajjar & Adil, 2010).

Metaheuristics are advanced techniques or algorithms that direct more effective and efficient exploration of the search space by lower-level heuristics. They are used to locate, produce, or choose a heuristic (partial search algorithm) that might offer an adequate solution to an optimization problem. This is particularly useful when dealing with large, complicated problems that are unsolvable using conventional approaches.

Metaheuristics uses techniques to break free from local optima, in contrast to simple heuristics that frequently become stuck there and limit their search to a wider range of options. Metaheuristics are especially effective at addressing challenging optimization issues because they can avoid local optima and completely explore the solution space.

Several heuristics and metaheuristics are used in SSAP, including a greedy heuristic and genetic algorithm(Urban, 1998), hybrid heuristics and metaheuristics (Marshall et al., 2014; Castelli &, Vanneschi, 2014), hyperheuristics including simulated annealing (Borin et al., 1994), reduced variable neighborhood search-based hyperheuristic (Yu et al., 2020). By pointing out the shortcomings of Yang and Chen's (1999) comprehensive model, Bai (2010) presented a non-linear shelf space allocation model (NLSSAM) for this problem. To tackle NLSSAM, he created a number of meta- and hyper-heuristics (Bai ,2010).

A genetic algorithm (GA) for shelf space allocation problems with shelf segments that can be increased or reduced was developed by some researches (Czerniachowska et al.,2021a; Czerniachowska et al.,2021b; Czerniachowska, 2022). The general GA has been enhanced with an improvement procedure conducted in the crossover and mutation phases, featuring solution improvement methods. These methods aim to identify less profitable products on the current shelf, reallocating them to shelves where they can generate more profit, and vice versa. This procedure facilitates high-quality product on shelves movement, thereby improving overall profitability (Czerniachowska et al.,2021a; Czerniachowska et al.,2021b; Czerniachowska, 2022) This increased agility in decision-making is likely to enhance overall operational efficiency. Consequently, these improvements can drive higher profits by optimizing the effectiveness of advertising investments and better aligning them with sales outcomes.

Several integrated models have been presented in recognition of the strong relationship between shelf space allocation and other retailing challenges. For instance, the model by Borin et al. (1994) can offer a solution for both product selection and shelf space distribution at the same time. For the suggested model, a simulated annealing approach was employed. Bai et al. (2013) explored the impacts of shelf space on demand while modelling shelves as two-dimensional spaces. Furthermore, the authors suggested a hyper-heuristic approach that is effective in solving this two-dimensional planogram issue (Bai et al.,2013). Urban (1998) combined a conventional shelf space allocation model with inventory control and product selection. A genetic algorithm was presented to tackle the problem. Nevertheless, there are often a lot of parameters in these models.

Heuristics, metaheuristics and hyperheuristics provide a flexible and robust approach to solving optimization problems, balancing exploration and exploitation of the search space. They are adaptable to a wide range of problems and can often be tailored to specific problem characteristics, making them a valuable tool in the field of optimization.

The optimization extends beyond mere placement — it encompasses pricing strategies, inventory management, and promotional tactics. It’s a holistic approach to retail optimization that seeks to use the full potential of every product on the shelves. So, in essence, in the art of retail merchandising every product placement, promotion, and pricing decision contributes to the increase of profitability.

B. Heuristics in media planning

Television advertising stands as a pivotal component within the television industry, offering advertising agencies a swift and captivating avenue to connect with potential buyers. For TV networks, the primary objective revolves around crafting advertisement schedules that meet advertisers’ requirements while simultaneously maximizing revenue streams. This challenge can be examined from various angles, with different perspectives emphasizing specific aspects that influence decision-making.

From the standpoint of TV networks, the paramount focus lies in maximizing ratings, particularly within target demographic segments. Conversely, advertisers prioritize understanding whether TV networks achieve anticipated ratings, which is crucial for assessing the effectiveness of their advertising campaigns.

There has been some work done on media planning from the standpoint of the advertising agency. For instance, Mihiotis and Tsakiris (2004) concentrated on the best time to place an advertisement within a TV program to increase the number of viewers. They discussed media strategy from the viewpoint of the advertising agency, but exclusively for that one particular product (Mihiotis &Tsakiris, 2004). A. By focusing on this element, they aimed to demonstrate how strategically crafted advertisements could significantly boost viewer numbers.

Obviously, the high-quality content in TV advertisements impacts audience engagement. The innovative storytelling and visual appeal could capture viewers’ attention more effectively. It could be emphasized that the importance of aligning ad content with viewers’ preferences results in maximizing reach and viewer retention.

García-Villoria and Salhi (2015) focused on the issue where fulfilling the requests of the advertisers is crucial because it might boost the TV channel’s revenue. The authors conducted an investigation into the issue of scheduling commercial ads in the television sector. Every advertiser’s client requested that the same brand advertisement be broadcast as many times as feasible throughout a predetermined window of time. Additionally, requests for audience ratings might be considered while scheduling. Two mixed integer linear programming models were created by García-Villoria and Salhi (2015). Additionally, two constructive strategies are suggested by García-Villoria and Salhi (2015): simulated annealing techniques and local search protocols (García-Villoria & Salhi, 2015).

Maximizing reach in TV media planning involves ensuring that the advertisement is seen by the largest possible audience within the target demographic. This is often measured using Gross Rating Points (GRPs), which represent the total exposure of an ad campaign by multiplying the percentage of the target audience reached by the frequency of exposure.

Fleming and Pashkevich (2007) concentrated on a media planning issue from the advertising agency’s point of view as well. In order to jointly maximize reach or GRP for the different brands while taking into account budgetary, competitive, and scheduling restrictions, their formulation entails choosing which commercial breaks to display the advertising of various companies (Fleming & Pashkevich, 2007).

Czerniachowska (2019) focused on the problem from an advertiser’s point of view maximizing viewership within budget limitations and airing frequency constraints. She developed a genetic algorithm and a set of low-level heuristics to tackle this issue (Czerniachowska, 2019).

Audience rating, another crucial metric, indicates the percentage of the total potential audience that views the program, helping advertisers select the most effective time slots and programs for their ads.

García-Villoria and Salhi (2015) included audience rating requests in a formulation of the TV scheduling issue that was single-objective. Every break has a corresponding audience rating (high, medium, or low) in their formation. Then, during commercial breaks with high or medium (or greater) audience ratings, the advertiser (or advertising agency) wanted to run the commercial a minimum number of times. They also included standards for airing regularity, which state that the same commercial must be aired as often as feasible over a predetermined period of time (García-Villoria & Salhi, 2015).

By strategically planning based on reach, GRP or audience rating metrics, advertisers can optimize their media spend to achieve the highest possible impact and engagement from their target audience. Additionally, these metrics allow advertisers to adjust their campaigns in real-time, ensuring that they can respond to changing viewer behaviours and preferences effectively. This dynamic approach helps maintain the relevance and effectiveness of TV advertising, ultimately driving better return on investment (ROI) for marketing investments.

According to Fleming and Pashkevich (2007), the TV network, the advertising agency, and the firm are the main participants in the media planning process. The following is how they explain the media planning procedure. Usually, businesses set a specific advertising budget and employ advertising agencies to handle media planning tasks on their behalf. The advertising firm is in charge of purchasing TV network advertising time on behalf of several businesses. The advertising agency then designs the best possible schedule for the commercials, deciding which shows and commercial breaks to use, as well as how frequently to run them. Maximizing viewership for the various brands is the primary objective (Fleming & Pashkevich,2007).

Advertisers often impose specific requirements regarding the scheduling of these advertising spots. Television networks must strike a delicate balance between airing programs and advertisements to retain their target audience while meeting the obligations outlined in the agreement. Furthermore, television networks must adhere to government regulations governing the broadcasting of advertisements.

In the past, choices on the distribution of advertising have typically been made at a macro level, involving the determination of the overall budget and then the distribution among media channels. The decision of which particular clients to target with advertising is currently made at a micro level due to the quick and continuous expansion of digital media. Ad response, cost per medium, and discount rate are all made possible in the macro situation by the effective framework that optimal control theory offers for maximizing company profit, even in the face of several rival brands. Optimal control theory, however, has never been used in the context of microtargeting specific clients. As a result, Danaher (2023) in his study demonstrated how optimum control theory can be modified for use with specific clients (Danaher ,2023).

So, in this world of television advertising, the set of channels forms the backdrop against which the auctions of commercial selling unfold. And within each channel, the breaks for TV advertisements serve as gateways to consumer engagement, connecting together the threads of entertainment and commerce into a maintenance of television marketing campaign.

Television advertising remains a powerful media channel due to its extensive reach and ability to deliver visually compelling and emotionally engaging content to a wide audience. It excels in creating strong brand awareness and recall, as its combination of sight, sound, and motion captures viewer attention more effectively than many other media.

The usefulness of each media channel and its associated cost are critical factors in devising an effective advertising strategy. Despite its high cost, television has historically proven to be highly successful due to its unparalleled reach and ability to create powerful, memorable visual narratives that resonate with a wide audience (Danaher ,2023).

TV ads can tell detailed and impactful stories, making them particularly useful for brands aiming to establish a deep connection with consumers. Moreover, television's broad demographic reach ensures that ads can target diverse segments of the population simultaneously, maximizing the impact of marketing campaigns.

West et al. (2014) offered an overview of the techniques American advertisers employ to determine their budgets for advertising and promotions, as well as the influences of risk, culture, and organizational experience on these decisions. Results imply that heuristics serve as a check on analytically based budgeting techniques and may also assist managers in managing risks. Gaining insight into the function of heuristics in budgeting is the first step towards streamlining the budgeting process for advertising and promotions (West et al., 2014). Åstebro and Elhedhli (2006) looked at the expert decision heuristics that predict the eventual commercialization of early-stage businesses. Åstebro and Elhedhli (2006) examined (1) the real experts' decisions using them as subjects and (2) the decision-making context involved careful consideration of each judgment (Åstebro T, Elhedhli S (2006).

By breaking down the advertising budget choice into four distinct components — baseline expenditure, adaptive experimentation, advertising-to-sales ratio, and competitive parity — Kolsarici et al. (2020) examined the role that heuristics and analytics play in this process. They suggested a method for calculating and deducing these four components' weights. Using this approach to analyze sales and advertising data from eight different brands in three different categories demonstrated consistently across all brands that managers deviate from optimality through adaptive experimentation, which is consistent with dual control theory's suggestion that they do so to discover the efficacy of advertising. The adaptive experimentation finding, together with evidence on the use of heuristic methods, implies that budget decision-making is characterized by bounded rationality (Kolsarici et al., 2020).

The insightful analysis of the complexities of advertising budget decisions is very important. By deconstructing the budget into definite key components and analyzing their respective impacts, an understanding of the usage of heuristics and other analytical tools applicable to managerial decisions could be obtained. Obviously, the managers could deviate from selected strategies through adaptive experimentation based on their experience.

Heuristics can help managers manage risks and serve as a check on other analytically based budget recommendations. Recognizing the importance that heuristics play in budgeting is the first step toward a much-needed process improvement in marketing budgeting (West et al., 2014). Heuristics also are applicable to the practical challenges of achieving perfect decision-making in media planning issues. Moreover, the reality of rationality in budget distribution for marketing campaigns, emphasizes the need for a balanced approach that incorporates both data-driven insights, heuristics techniques and adaptive learning.

Furthermore, the development of analytics and the abundance of data need the adoption of more evidence-based budgeting procedures in order to balance the dominance of heuristic norms (Danenberg et al.,2016). Specifically addressing the problem of figuring out the total advertising budget, Kolsarici et al. (2020) highlight two general approaches: heuristics (such as the advertising-to-sales ratio) and analytics (such as profit maximization). The second big choice is how to divide the entire budget among the various mediums (Kolsarici et al., 2020).

Television's capacity to engage viewers emotionally and reinforce brand messages through repeated exposure makes it a valuable investment, particularly for brands seeking broad market penetration and significant impact. Additionally, TV advertising often benefits from being a part of cultural conversations, further amplifying its effectiveness and value.

From the standpoint of an advertising agency, Evangelista and Regis (2020) focused on a multiobjective optimization problem in TV advertising. This problem involves choosing which commercial breaks to air the ads of different brands in order to jointly maximize reach or gross rating point (GRP) for the different brands while taking budget, brand competition, and other scheduling constraints into consideration. Evangelista and Regis (2020) formulated this problem in terms of multiobjective integer programming, created and put into practice algorithms that produce provably Pareto-optimal solutions. In order to help a decision maker select appropriate subsets of the generated Pareto-optimal solutions, Evangelista and Regis (2020) additionally proposed reduction and visualization processes (Evangelista & Regis, 2020).

An auction is usually held to sell the spots because the demand for them is typically higher than the supply (Alaei & Ghassemi-tari ,2011; Ghassemi- tari & Alaei ,2013). The TV station must choose which bids to accept in order to maximize its earnings rather than identifying the spots to be purchased (Kimms & Müller-Bungart ,2007). The collaboration between TV networks and advertisers involves negotiating terms to ensure the effective delivery of advertising messages to the desired audience while adhering to regulatory requirements. This symbiotic relationship is crucial for both parties to achieve their respective goals and maintain a successful partnership in the competitive television industry.

Moreover, collaboration and communication among stakeholders are crucial for successful media planning initiatives. TV stations, advertising agencies, and advertisers must work in tandem to align their goals, negotiate terms, and optimize the allocation of advertising inventory.

In summary, television media planning involves a complex interplay of stakeholders, objectives, and constraints. By developing strategic media plans that balance the interests of all participants, advertisers can maximize the impact of their campaigns, reach their target audience effectively, and drive measurable results within budgetary constraints.

III. Problem definition

A. *Knapsack problem*

The knapsack problem (KP) serves as a quintessential illustration within the domain of combinatorial optimization challenges. Its core objective revolves around optimizing the selection of items intended for inclusion in a knapsack while adhering to the constraints imposed by its weight capacity. Conceptually, this problem entails a collection of items, each characterized by a specific weight and corresponding value. Simultaneously, there exists a knapsack awaiting its load, with a predetermined maximum weight it can accommodate.

Scholars have examined the knapsack problem since 1897. The issue was initially brought up by researcher Tobias Dantzig. The moniker that is gained in mythology before a mathematical issue is completely specified was proposed by Dantzig. Capable reduction techniques have been developed for all knapsack issues, allowing one to fix many choice variables for objective functions (Pisinger ,1995). There exist several variants of knapsack problems, including but not limited to multiobjective, multidimensional, quadratic, and subset-sum issues (Binguler et al.,2015).

Here's the setup: A set of items N $(i = 1, ..., N)$ is defined, denoted by an array, where each item possesses an individual weight (w_i) and value (v_i). Meanwhile, the knapsack is poised for filling, constrained by a strict weight limit (W). The objective encompasses two primary facets: firstly, to ascertain the items to be placed into the knapsack in a manner that their combined weight does not exceed the knapsack's capacity, and secondly, to maximize the total value of the items accommodated.

The knapsack problem can be likened to a strategic packing endeavour, wherein each item selection necessitates a nuanced evaluation of its value contribution juxtaposed with the space it occupies within the knapsack (Bai ,2010; Pisinger ,1995; Martello & Toth ,1987; Pisinger ,1999; Connolly et al.,1991).

Maximize

$$\max \sum_{i=1}^{N} v_i x_i \tag{1}$$

subject to the constraint:

$$\sum_{i=1}^{N} w_i x_i \leq W \tag{2}$$

where

$$x_i \in \{0,1\} \tag{3}$$

B. *Shelf space allocation*

In the retail sector, there exists a challenge regarding the placement of products on store shelves to maximize profitability through sales. Frequently, retailers employ planograms as a strategy to address this objective.

A planogram is a graphical depiction or schematic representation outlining the arrangement of merchandise on retail shelving or in-store displays, aimed at enhancing sales performance and improving the overall shopping encounter.

Essentially, it functions as a detailed blueprint for the spatial organization of goods, incorporating considerations such as the strategic placement of items, product variety, spatial allocation, and promotional elements. Widely employed by retailers, planograms are instrumental in ensuring that shelf layouts effectively engage customers, emphasize key products, and facilitate ease of navigation.

The planogram, comprising shelves, necessitates the precise placement of a variety of products, with each product assigned to a specific shelf. Subsequently, determining the quantity of each item within predefined permissible ranges becomes the issue. The overarching objective is to optimize the planogram layout to maximize the cumulative profit derived from the sale of products.

Retailers must transcend the mere task of shelf replenishment; they must strategically orchestrate the planogram layout to ensure that every square inch of retail space contributes to revenue generation. The arrangement and the number of items of each product are not unsystematic but rather a meticulously calculated approach aimed at optimizing return on investment.

This intricate process involves considerations beyond mere product placement. Factors such as product visibility, consumer behaviour, seasonal variations, and promotional strategies all play pivotal roles in determining the effectiveness of the planogram. Therefore, achieving optimal profitability necessitates a holistic approach that integrates both quantitative analysis and qualitative insights into consumer preferences and market dynamics. In this dynamic retail landscape, elements such as product positioning, visibility, and consumer behaviour exert significant influence. Whether through attention-grabbing presentations or tactical product bundling, each decision is driven by the overarching objective of enhancing sales performance and augmenting profitability.

While additional constraints within the retail realm exist, our current research we do not focus on them.

C. Advertisement allocation in media planning

In the realm of television media planning, the focus of this research is on the intricacies of the advertisement allocation problem (AAP). This problem space involves multiple stakeholders, including TV stations, TV channels, advertising agencies, advertisers, and viewers who serve as potential consumers of the advertised products. The primary objective is to develop optimal media plans that effectively boost the target demographic ratings while minimizing costs, all while adhering to the constraints of each participant.

At its core, the AAP revolves around the strategic allocation and scheduling of TV advertisements to maximize the impact on the desired audience. TV networks strive to optimize their profits by selecting and scheduling advertisements tailored to the target demographic. On the other hand, advertisers seek to maximize the reach of their products within the confines of their advertising budget.

Within media planning, a critical challenge arises in strategically positioning commercials within television breaks to maximize viewership. Across the diverse array of channels, these TV commercial breaks serve as crucial connectors facilitating the connection between advertisers and viewers. They’re not just interruptions of TV programs, but they are opportunities to captivate, persuade, and leave a lasting impression in the minds of audiences.

There is a set of TV channels, and each of them is a distinct window into the world of entertainment, news, and information. Within this realm, on each TV channel, breaks for TV commercials punctuate the flow of content, offering businesses and advertisers a platform to showcase their products and messages to a target audience. So, advertisers can buy parts of TV breaks for their commercials.

Typically, a television break is subdivided into a predetermined number of advertising spots, each available for purchase at varying prices and possessing its own viewership and demographic metrics. The primary objective entails managing the placement of these advertisements in a manner that maximizes viewership while operating within the budget constraint set by the advertiser.

While various constraints may exist, our current research does not focus on them. Instead, the scale of an advertising campaign is quantified by considering factors such as budget allocation, spot viewership metrics, and viewing frequency over a specified duration, such as two weeks or one month. The allocated budget specifically pertains to the portion earmarked exclusively for spot purchases, excluding expenditures associated with market research, pre-testing, post-testing, and professional fees.

The efficacy of an advertising campaign can be assessed using several key parameters. These metrics are delineated on a daily, weekly, or monthly basis, contingent upon the scope and duration of the experiments conducted.

Gross Rating Point (GRP) serves as a metric describing the overall audience reach of a commercial. It can be evaluated both in relation to the entire audience and specific target demographics. Essentially, it represents the percentage of the target audience that encounters the commercial. Notably, GRP does not provide information on the frequency of individual exposure to the commercial, but rather serves as a measure of weight rather than effectiveness. The calculation of GRP involves aggregating the ratings of all instances of commercial airing throughout the entire advertising campaign (Araman & Popescu ,2010; Araman & Popescu ,2011; Friedman ,1971; Guerriero, 2017).

On the other hand, Reach quantifies the number of individuals within the target audience who have been exposed to the commercial a specified number of times. Higher reach indicates that more potential buyers have encountered the commercial. While the ideal reach would be 100%, signifying that the entire target audience has been reached, achieving this is often impractical. Reach calculations consider each recipient of the advertisement only once, regardless of the frequency of exposure to the TV commercial. Reach can be expressed in various units, including individual counts, thousands, millions, or as a percentage of the target audience (Friedman, 1971)

Reach can be defined in two ways: as the portion of the target audience that has viewed a commercial a specific number of times or as the portion that has seen the commercial at least a certain number of times within a given time frame, such as a month or a week.

In the context of mass media, the concept of rating is integral to calculating reach. Rating refers to the percentage of individuals within the target audience who have had the chance to view the commercial. Essentially, it represents the viewership of the specified channel within a defined time period and can be expressed either in numerical units or as a percentage.

Frequency, or average viewing opportunity, is another important metric that can be computed by dividing the total rating (GRP) accumulated during the advertising campaign by the reach achieved through advertisement placement (Friedman, 1971).

The advertiser's objective in addressing the commercial allocation problem is to maximize viewership. Within this framework, viewership may be quantified using metrics such as GRP, reach, frequency, or any other relevant rating system.

IV. Mushroom picker heuristics description

In this study, we propose innovative mushroom picker heuristics designed to tackle knapsack-like resource allocation problems incorporating product categorization. Our methodology involves the creation of multiple unique heuristics variants, each distinguished by a specific sorting order for assembling solution components. The fundamental methodology of our heuristics comprises several essential steps:

1) Initialization - Preparing the forest ground: The process begins by initializing the algorithm and defining the problem parameters for the creation of the forest clearings. Envision the green forest landscape, where mushrooms grow with diverse heights and densities, presenting a rich and varied solution space ripe for exploration and optimization.
2) Solution assembly - Identifying mushroom clearings: The heuristic constructs potential solutions. Determine which forest fields are the subject of attention when picking mushrooms. These demarcated areas constitute the focal points for analysis and assessment, serving as the canvas for our comparison and evaluation processes.
3) Selection - Picking the mushrooms: The solutions are selected based on some criteria and the designated sorting order. Engage the picker in picking mushrooms and putting them into the basket. To define which mushrooms should be picked, use specific parameters. These parameters must be finely tuned to deal with the intricacies of the task at hand, ensuring accuracy and efficiency in the execution of our objectives. These parameters include:
   - Length of picker's movement path: Control the distance that the picker walks through the forest while collecting mushrooms.
   - Target mushroom size (height, weight): Specifies the size (height, weight) below which the mushrooms are not collected by the picker, guiding the picker's attention to more profitable mushrooms.
   - Picker's basket size: Determines the capacity of the basket, which allows the picker to collect the number of mushrooms during his walk in the forest. Therefore, the picker is only interested in mushrooms that are profitable enough. There is no goal to fill in the basket as fast as possible. Hence, the picker does not collect all the mushrooms that he sees. The goal is to find the biggest mushroom.
4) Evaluation: There are no random elements of solution generation. Each generated solution satisfies all constraints. Therefore, each generated solution undergoes evaluation based on specified metrics such as profitability or resource utilization. Not all mushrooms from the selected clearings are picked and collected in the basket. Only better or at least no worse than already in the basket mushrooms are collected.
5) Refinement: The algorithm refines selected solutions through iterative improvements or adjustments, aiming for optimization. In other words, the above-mentioned tuning parameters are corrected, and the picker repeats picking the mushrooms. Because there is no random solution generated or parameter selection, the number of iterations is very small and is based on previous attempts to find profitable solutions. Parameters improve the previous experience.
6) Termination: The process concludes upon reaching a predefined stopping criterion, such as a specified number of iterations of reducing solution space parameters setting, achieving a satisfactory solution quality or achieving the number of good solutions (which are selected to the basket).

By employing these mushroom picker heuristics, we aim to propose a versatile and effective approach for addressing complex resource allocation problems, particularly those characterized by knapsack-like structures and product categorization requirements. Item categorization simplifies the parameter tuning because each parameter is used for a category and not for a single item.

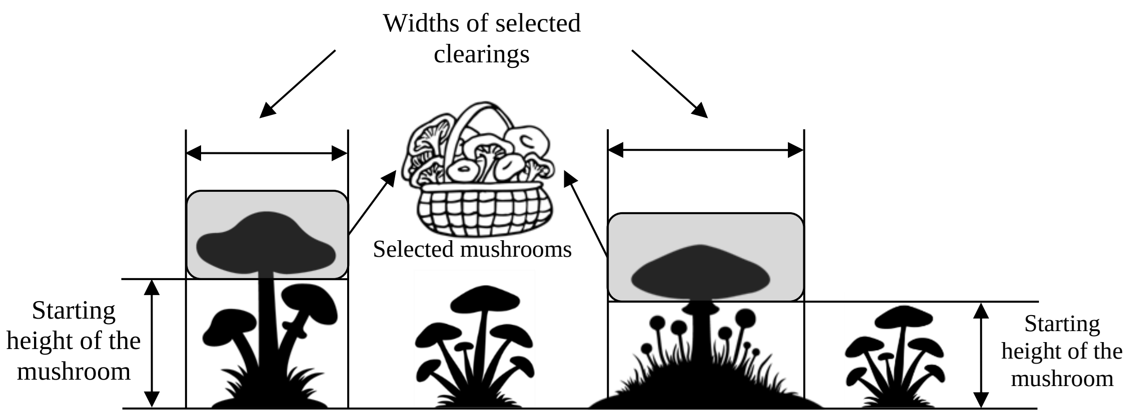
Fine-tuning of these parameters enables the creation of a broad spectrum of solutions, fostering an exhaustive exploration of the solution space. This versatile approach facilitates thorough investigation and analysis, offering insights into the diverse possibilities within the solution space.

Unlike a real mushroom picker whose goal is to fill the basket with mushrooms, our picker is interested in finding the biggest mushroom without collecting the mushrooms all over the forest. Each mushroom represents the solution and not the product in the knapsack problem. The basket does not represent the knapsack. The basket stands for that case that only a definite solution number could be created and investigated, and no more solutions. Hence, the picker collects only high-quality solutions and puts them in the basket. Moreover, when the picker intends to put the next mushroom into the basket, he first compares it to the mushrooms already in the basket. If the mushroom is smaller, he does not take it. Among the mushrooms in the basket, the biggest mushroom (the solution with the highest profit) was selected.

By integrating the above-explained steps, our methodology constructs a structured framework for tackling the knapsack-like resource allocation problem. We fill in the basket of the limited size with mushrooms of a definite size and profit. However, our main goal is to find the biggest mushroom without exploiting the whole forest. Drawing inspiration from the metaphor of mushroom picking, our approach offers an efficient means to address the complex problem, enabling the identification of solutions that are either optimal or at least near-optimal. This systematic integration of steps enhances the effectiveness of our approach, facilitating a streamlined and insightful exploration of the problem space.

Figure 1 illustrates that there are two clearings in the forest where mushrooms are selected. Each mushroom represents a solution. Each clearing has a height set starting from which the mushrooms are picked by the picker and placed in the basket. The heights may differ in different clearings. The width of clearings also may differ. The mushrooms in other clearings are not under the attention of the picker, even if their height exceeds any of the starting heights on the selected clearings. Therefore, the goal is to select the clearings with big mushrooms and not omit any of the profitable clearings. The destinations between the selected clearings are not important. The algorithm execution time for solution generation, specifying which solution is taken, is calculated only on the selected clearings where the picker works.

FIGURE 1. LOOKING FOR FOREST CLEARINGS TO COLLECT MUSHROOMS



Let us shed light on the practical application of the proposed heuristics. The iterative process entails identifying specific clearings of the forest where bigger mushrooms grow and refining the parameters of the clearings, examining to maintain a manageable picking duration, all while ensuring that collected mushrooms do not surpass the capacity of the picker's basket. As a reminder, our primary goal is to pinpoint the biggest mushroom within the forest clearing.

Figure 2 elucidates the notation of mushroom-picking heuristics in a manner applicable to practical scenarios. Initially, the task entails estimating the total number of solutions and either generating all feasible solutions or selecting a subset of them when dealing with large problem sizes where exhaustive solution generation is impractical.

Subsequently, the process involves surveying the forest clearing to identify areas with mushrooms necessitating picking. This entails a meticulous evaluation of mushroom size throughout the clearing, identifying patches where the mushroom has exceeded the desired size. By discerning these regions, we can concentrate the picker's efforts on the most lucrative areas. In essence, the objective is to generate solutions within specified parameters rather than exhaustively exploring all possible options. For instance, in the context of the knapsack problem with categorized items, this may entail defining the range of space allocated for each category, encompassing minimum and maximum space constraints.

Once we've delineated the clearings requiring picking, it becomes obligatory to fine-tune the parameters of the mushrooms to be picked to optimize their efficiency while maintaining effectiveness. This involves adjusting variables such as the length of the pickers' movement, the desired size of the picked mushroom, and the capacity of the picker's basket. These adjustments play a pivotal role in striking a balance between picking precision, time efficiency, and the storage capacity of the picker's basket.

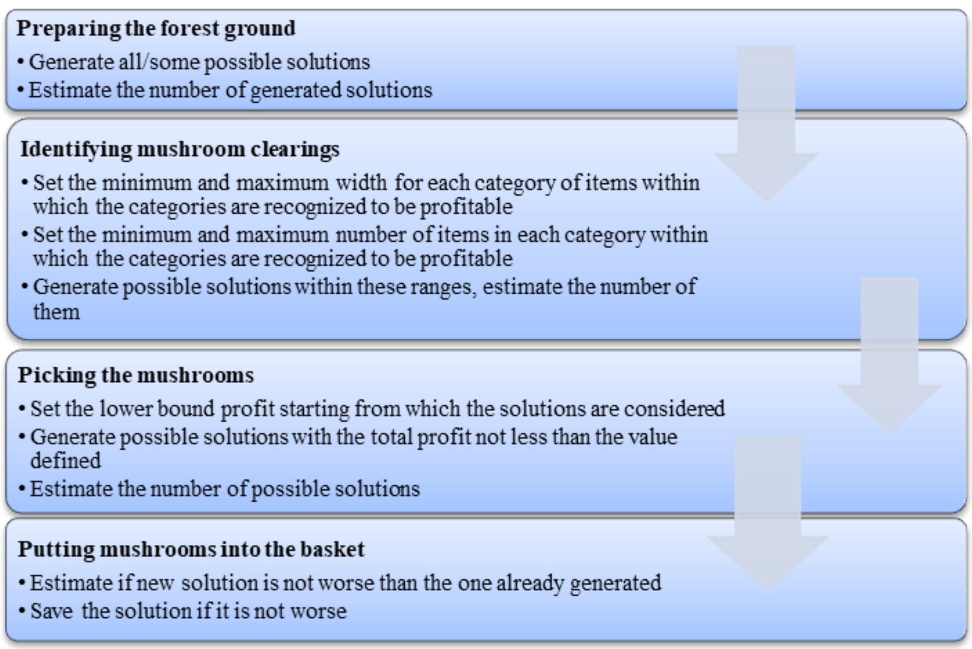
Subsequently, we proceed to estimate the profitability associated with solutions falling within the specified range of space and the range of item quantities. We establish a threshold for the total profit below which solutions are deemed unsuitable. This ensures that selected solutions are guaranteed to yield profits surpassing a predefined threshold.

Nevertheless, it is essential to ensure that the overall picking duration remains within acceptable thresholds. Excessive time expense could retard overall productivity. Therefore, at each stage subsequent to parameter setting, we assess the potential number of feasible solutions. If this quantity proves excessively large and cannot be generated within a reasonable timeframe, we adjust tuning parameters accordingly. For instance, this might involve narrowing the category width range, reducing the variance of item quantities within each category, or augmenting input profit so that the solutions with profits below are not considered. By optimizing the picker's moving parameters, our aim is to strike a balance where solution generation is expedited while ensuring satisfactory total profit.

Additionally, we must consider the capacity of the picker's basket receptacle to accommodate picked mushrooms. The number of solutions generated and investigated corresponds to the number of mushrooms in the basket and consequently the size of the picker's basket. Among these solutions, the one yielding the maximum profit is selected. Hence, it is crucial to closely monitor the number of picked mushrooms (i.e., estimate the number of solutions at each tuning step) and adapt our picking strategy (i.e., tuning parameters) as necessary to prevent overflow.

Moreover, at each step of picking subsequent mushrooms, the picker could estimate if the mushroom is bigger or at least not worse than the mushrooms inside the basket. The picker does not put mushrooms in the basket mushroom, which is worse than mushrooms already in the basket. He does not compare the new mushroom with each of the picked mushrooms, but he quickly estimates if it is at least not worse.

FIGURE 2. A GLANCE AT CONSOLIDATED MUSHROOM PICKING HEURISTICS STEPS



V. Application of Mushroom Picker Heuristics to Real Problems

A. *Application of mushroom picker heuristics to shelf space allocation problem*

In the context of the shelf space allocation problem (SSAP), two decision variables come into play:

- Binary variable: Indicates whether a product is allocated to a shelf.
- Positive integer variable: Specifies the quantity of stock-keeping units (SKUs) of each product assigned to each shelf.

Consequently, the solution comprises a sequence of numbers:

- Shelf allocation: A sequence of binary numbers (0/1) indicating the presence or absence of a product on a shelf.
- Product allocation: A sequence of integer numbers representing the quantity of SKUs allocated to each shelf for a given product.

Preparing the forest ground:

1) Develop a series of shelf allocation sequences that adhere to constraints related to the allocation of products on individual shelves. These constraints may include specifications regarding which shelves are permissible for product placement, which products can be co-located on the same shelf, and whether certain products must or must not be positioned adjacent to each other.
2) Generate a collection of shelf allocation sequences that conform to constraints associated with the allocation of products across multiple shelves. These constraints may encompass factors such as the permissibility of neighbouring shelves, the allowance for products to span multiple shelves, limitations on the number of shelves allocated, and requirements for products to occupy distinct shelves.
3) Establish a set of product allocation sequences corresponding to each generated shelf allocation sequence, taking into account constraints pertinent to product placement on shelves. These constraints may involve

considerations such as minimum and maximum SKU quantities, dimensions of the shelves (e.g., length, depth, height), and the visibility of products relative to neighbouring SKUs.

4) Filter out product allocation sequences from the generated set that fail to meet specified criteria, such as exceeding product limits when allocated across multiple shelves, adhering to category constraints (e.g., minimum category size, ensuring visible category boundaries across multiple shelves, SKUs that fail to form hierarchical rectangular shapes of categorization on several shelves, etc.), and adhering to grouping constraints if products are categorized.

Once the groundwork of establishing the solution space related to preparing a forest ground is complete, we gain the ability to gauge the potential number of shelf and product allocations. This step is pivotal for gauging the workload's manageability and assessing the generated solutions within an acceptable timeframe. Essentially, it's akin to surveying the lawn to determine where the mushrooms are bigger. By conducting this estimation, we're empowered to streamline our focus on generating a subset of sequences that promise effective solutions. In essence, we're pinpointing specific areas in the forest where the mushrooms are bigger. Moreover, by changing the minimum and maximum SKU values, we can approximate the profitability associated with a given sequence of product allocations.

This process allows us to measure the effort of solution generation, ensuring that we navigate and concentrate on sequences that hold the higher profit. Just as a picker tends to the areas where the mushrooms grow lush, we focus our attention on the solutions that yield the most favourable outcomes.

In order to implement the suggested mushroom picker algorithm efficiently, it is useful to correctly define specific steering parameters. These parameters play a crucial role in directing the execution of the algorithm and enhancing its effectiveness. They serve as guiding principles, steering the algorithm towards near-optimal or optimal performance by overseeing the generated shelf and product allocations.

Identifying mushroom clearings. These parameters concern steps 3 and 4 of the forest ground initializations.

In order to effectively optimize the allocation of products on shelves within each category, define two key parameters: the minimum category width (referred to as parameter 1) and the maximum category width (referred to as parameter 2). These parameters delineate the allowable range of widths for categories across all shelves within the category. By specifying these parameters, we establish clear boundaries within which the algorithm operates, ensuring that category widths conform to predetermined constraints. More profitable product category gets more space. All product allocations are not considered.

In order to refine the optimization process for product placement on shelves, define three additional parameters: the minimum number of products (referred to as parameter 3) and the maximum number of products (referred to as parameter 4) that can be placed on each shelf, profitable groups of products (referred to as parameter 5) on the same shelf. These parameters establish clear guidelines for the quantity of products to be allocated to shelves, regardless of the myriad permutations possible within these shelf allocations. More profitable products are placed with less profitable products on the shelves because more profitable products must get more space. The shelf allocation with different profitable products on the same shelf and different profitable products on other shelves is worse because, in this case, profitable products get less space than if they were placed on the shelf along or together with less profitable products. All shelf allocations are not considered.

It's important to note that our focus here is on establishing the range of allowable product quantities for each shelf rather than exhaustively considering every possible arrangement of products across shelves. This approach allows us to streamline the optimization process, focusing on the core parameters that govern product allocation within the given retail space.

By defining the minimum and maximum product quantities and the minimum and maximum category size, we ensure that the optimization process remains grounded within practical thoughts, striking a balance between efficient space utilization and maintaining an appropriate level of product numbers and product diversity on shelves. These parameters serve as vital restrictions, guiding the algorithm in its effort to optimize the space of product placement while adhering to predefined constraints and maximizing profit objectives.

It's important to understand that, in this context, we are solely concerned with determining the width range for categories on shelves, rather than delving into the specifics of product allocations within varying all possible category widths on whole shelf lengths. This focus allows us to streamline the optimization process, refining the parameters that set the layout of categories within the retail space.

Picking the mushrooms. At this point, the product allocations have been formed. We define the steering parameters the use of which can reveal the most profitable product allocations to form the solution.

1) Length of picker's movement path.

Taking into account the average width of product allocations within the designated category, it is useful to delineate the minimum (referred to as parameter 6) and maximum (referred to as parameter 7) category widths for each category. This ensures that product allocations falling outside the specified width range are excluded from analyzing in later steps.

Furthermore, by considering the number of product allocations present on each shelf, it becomes necessary to define the maximum number of product allocations (referred to as parameter 8) that will proceed to subsequent steps. Any additional product allocations beyond this maximum threshold are disregarded. Because we take only a part of product allocations that are formed, the sorting order must be applied for all product allocations to correctly identify product allocations that get into the defined set. It's essential to note that the sorting order may differ or remain consistent for defined variants of heuristics. The sorting underscores the importance of maintaining uniformity and coherence throughout the optimization process. Possible sorting rules for parameter 8 are:

- Rule 8.1: sort the product sequences in not descending order of category width, next, sort the parts inside sorted ones in not ascending order of category profit.
- Rule 8.2: sort the product sequences in not ascending order of category profit next, sort the parts inside sorted ones in not descending order of category width.

In this context we understand under category profit – the total profit of the product allocation of the given category; category width – the total width of the product allocation of the given category.

If, even after considering the maximum number of product allocations (referred to as parameter 8), the quantity of these allocations remains excessive, a supplementary step is necessary. Prior to applying parameter 8, it becomes essential to define grouping criteria (referred to as parameter 8). This entails selecting, for each total width, the product allocation yielding the maximum total profit. In essence, this grouping procedure ensures that only one product allocation is retained for each category width (considering all shelves). Subsequently, parameter 8 can be implemented to further refine the selection process.

Additionally, in light of the number of product allocations within each category, it is useful to establish the maximum allowable quantity (referred to as parameter 10) of product allocations for the category that will advance to subsequent stages. The rest of the product allocations are excluded from consideration. It's crucial to note that the sorting order varies depending on the variant of heuristics employed, underscoring its significance in maintaining consistency and efficacy throughout the optimization process. Possible sorting rules for parameter 10 are:

- Rule 10.1: sort the product sequences in not ascending order of profit, next, sort the parts inside sorted ones in not ascending order of profit ratio.
- Rule 10.2: sort the product sequences in not ascending order of profit ratio, next, sort the parts inside sorted ones in not ascending order of profit.

In this context, the profit ratio is calculated as the ratio of the total profit of the products allocated on all shelves divided by occupied space by the products (free space may be taken or not into calculations).

2) Target mushroom size (height, weight).

Taking into account the average profit that forms the product allocations within the designated category, it is necessary to define the minimum profit threshold for each category (referred to as parameter 11). Taking into account the sum of average profits of each category that forms the product allocations within the designated category, it is necessary to set the minimum total profit threshold for the future solution (referred to as parameter 12).

3) Picker's basket size.

The size of the picker's basket is the total number of solutions we can explore in the predetermined timeframe or the predetermined time for solution generation based on the technical resources. This encompasses factors such as computational power, algorithm efficiency, and time allocated for solution generation.

Find the biggest mushroom among the picked mushrooms in the basket.

Following the generation of a set of solutions, the subsequent phase entails identifying and selecting the solution that yields the highest total profit. This procedure ensures that the chosen solution maximizes profitability, representing the optimal outcome among the pool of generated solutions. This solution may not be globally optimal.

Additional notes for parameter usage.

The distinction between category width parameters 1 and 2 and category width parameters 6 and 7 lies in the point when they are applied in the explained steps. Parameters 1 and 2 take into account the length of the shelf, while parameters 6 and 7 are determined based on the average width of product allocations within the specified category.

Parameters 8 and 10 diverge in their definitions, with parameter 8 specifying the number of product allocations on the shelf, allowing for varying numbers across different shelves. In contrast, parameter 10 denotes the number of product allocations on all shelves within the category, allowing different values for different categories.

For parameters 1, 2, 6, 7, 10, and 11, distinct values may be assigned to different categories, reflecting the unique characteristics and requirements of each category. Conversely, parameters 3, 4, and 8 may vary across different shelves, accommodating specific considerations to individual shelves.

Parameter 12 reflects the total profit specifically for all categories and all shelves.

By providing above mentioned parameters, we streamline the optimization process, focusing solely on product allocations that align with the profitable solutions. Furthermore, these parameters serve as a means of quality control, allowing us to monitor and evaluate the outcomes of the algorithm's execution. Through systematic checks and adjustments based on these parameters, we can fine-tune the algorithm's behaviour, refining its performance over time.

B. Figure 3 summarizes the explained parameters Application of mushroom picker heuristics to media planning problem

In the context of the media planning problem, one decision variable comes into play:

- Binary variable: Indicates whether a commercial is emitted in the definite channel, in the definite spot of the break.

Consequently, the solution comprises a sequence of numbers:

- Spot allocation: A sequence of binary numbers (0/1) indicating the presence or absence of a commercial in the break on the channel.

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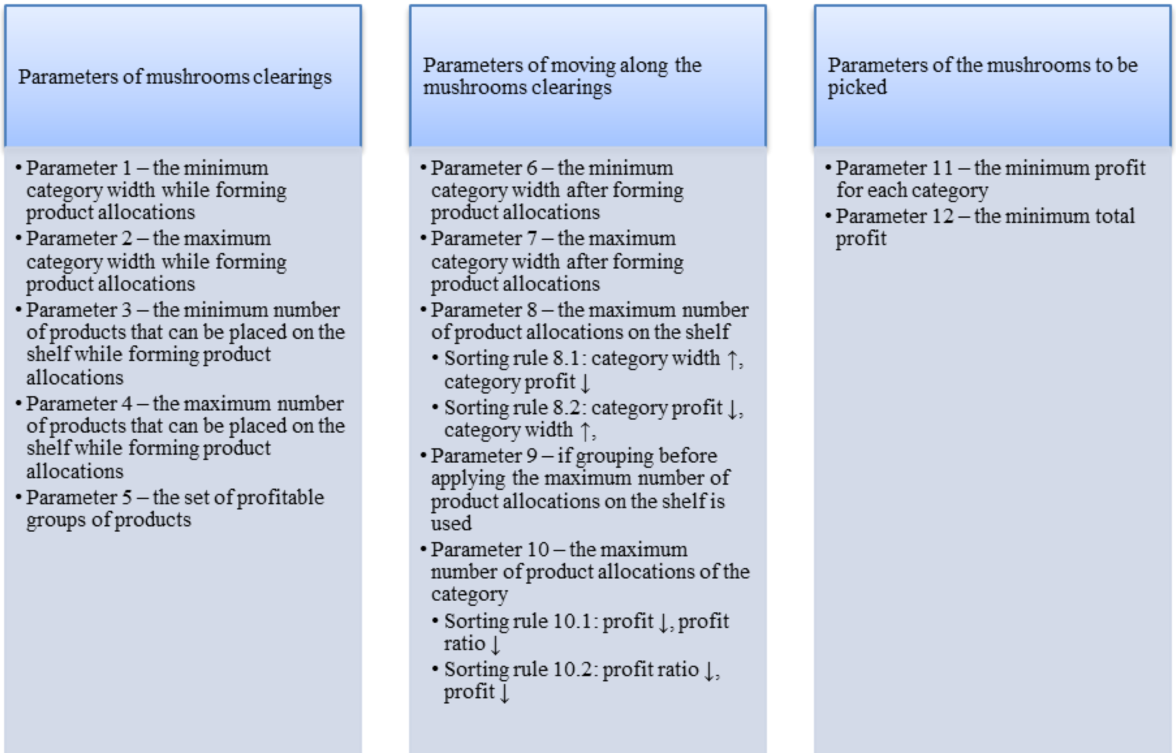
FIGURE 3. MUSHROOM PICKING PARAMETERS IN SHELF SPACE ALLOCATION PROBLEM



Preparing the forest ground:

1) Develop a series of spot allocation sequences that adhere to constraints regarding the placement of commercials on spots within a single channel. These constraints may encompass specifications on where commercials are permissible to be aired, which breaks or TV programs they can be featured in, whether certain commercials can be repeated within the same break or channel, proximity limitations between the same and similar commercials within a break, requirements for specific commercials to be positioned as first or last in a break, and the frequency of commercial broadcasts on the channel.
2) Generate a collection of spot allocation sequences that comply with constraints guiding the allocation of commercials across multiple channels. These constraints may include specifications on simultaneous airing across several channels, the number of channels on which the commercial must be broadcast, sequential airing across different channels following the initial broadcast, and the frequency of commercial broadcasts across diverse channels.
3) Exclude spot allocation sequences from the generated set that fail to meet specified criteria, such as exceeding commercial emission limits when broadcast on multiple channels, adhering to TV program constraints (e.g., program duration, break duration, visibility of commercials during channel switching), or conforming to grouping constraints based on TV topics or target audience demographics, and any associated restrictions on these topics and demographics.

After the initial work of establishing the solution space, akin to preparing a forest ground, is complete, we can assess the potential number of spot allocations. This step is crucial for managing workload and evaluating solutions within a reasonable timeframe. It's like surveying a forest to identify clearings with big mushrooms. By estimating this, we can focus on generating effective solutions akin to pinpointing lush areas of the forest. Historical data of viewing commercials in the definite spots of the break helps approximate the total viewership and frequency of viewing of each commercial. This process ensures we concentrate efforts on high-quality solutions, much like a picker tending to select mushroom clearings in the forest.

To efficiently implement the proposed mushroom picker algorithm, it's essential to accurately define specific steering parameters. These parameters are pivotal in guiding the algorithm's execution and improving its efficiency. Serving as guiding principles, they direct the algorithm towards achieving near-optimal or optimal solutions by overseeing the generated spot allocations.

Identifying mushroom clearings. These parameters concern steps 2 and 3 of the forest ground initialization.

To optimize the allocation of commercials in spots on each channel, two important parameters must be defined: the minimum budget (referred to as parameter 1) and the maximum budget (referred to as parameter 2) that the advertiser can pay for one channel. These parameters define the range of permissible budgets for all channels on which the advertiser accepts to emit the commercial. By establishing these parameters, clear boundaries are set for the algorithm's operation, ensuring adherence to predetermined constraints. Additionally, it's important to note that the placing of commercials to spots corresponds to the viewership of the TV program, TV break inside or during the TV programs, and not all spots on all channels to which commercials can be placed are taken into consideration.

To enhance the optimization process for commercials in spot placement, we introduce four additional parameters: the minimum number of commercial repetitions on the channel (referred to as parameter 3), the maximum number of commercial repetitions on the channel (referred to as parameter 4), the minimum interval measured in time or spot numbers before the next repetition of the same commercial is possible (referred to as parameter 5), the maximum interval measured in time or spot numbers after the next repetition of the same commercial is possible (referred to as parameter 6). These parameters provide clear directives for the quantity of commercial repetition emitted on the channels, irrespective of the potential permutations within these commercial emitting in all possible spots.

It is crucial to emphasize that our primary focus lies in delineating the permissible range of commercial repetitions on the same channel as well as on multiple channels rather than exhaustively examining every conceivable commercial to spot allocation across all channels. This strategic approach enables us to streamline the optimization process, concentrating on the core parameters that steer commercials to spot assignments within the designated TV breaks.

By establishing the minimum and maximum commercial repetitions, as well as the minimum and maximum spent budget on the channel, we ensure that the optimization process remains anchored in practical considerations. This approach strikes a delicate balance between efficient utilization of budget and maintaining an optimal level of commercials on channel emission. These parameters serve as crucial inputs, guiding the algorithm in its endeavour to optimize commercial emitting while adhering to predefined budget limitations and maximizing viewership objectives.

It is essential to recognize that, in this context, our primary concern is defining the channel's budget range for the commercial rather than intricately examining all potential commercial to spot placement across various channels' budgets, spanning the total budget that the advertiser agreed to spend for the advertising campaign. This focused approach enables us to streamline the optimization process, focusing on steering parameters.

Picking the mushrooms. At this point, the commercials to break spots have been assigned to all channels. We define the steering parameters the use of which can help to find variants with the highest viewership to form the solution.

1) Length of picker's movement path.

Considering the average cost of commercials to spot placements on each channel, it is beneficial to define both the minimum (referred to as parameter 7) and maximum (referred to as parameter 8) budget that can be spent on each channel. This ensures that commercials outside the specified budget range are disregarded in subsequent analyses.

Moreover, considering the number of commercials-to-spot placements on each channel, it becomes useful to establish the maximum number of commercials-to-spot placements (referred to as parameter 9) that will proceed to subsequent stages. Any surplus commercials-to-spot placements beyond this designated threshold are disregarded. Given that only a subset of commercials to spot placements is retained, applying a sorting order is necessary to ensure accurate identification of the commercials to spot placements included in the defined set. It is noteworthy that the sorting order may vary or remain consistent across different variants of heuristics. The implementation of sorting rules is essential for maintaining uniformity and coherence throughout the optimization process. Possible sorting rules for parameter 9 include:

- Rule 9.1: sort the commercials to spot placements in not descending order of spent budget considering all channels, next, sort the parts inside sorted ones in not ascending order of viewership.
- Rule 9.2: sort the commercials to spot placements in not ascending order of viewership considering all channels, next, sort the parts inside sorted ones in not descending order of spent budget.

If, despite considering the maximum number of commercials to spot placements (referred to as parameter 9), the quantity of these placements remains excessive, an additional step is needed. Before applying parameter 9, it becomes necessary to set grouping criteria (referred to as parameter 10). This involves identifying, for each spent budget considering all channels, the commercials-to-spot placements with the highest viewership. Essentially, this grouping process ensures that only one commercial-to-spot placement is retained for each budget, encompassing all channels. Following this, parameter 9 can be introduced to further fine-tune the selection process.

Moreover, considering the quantity of commercial-to-spot placements on each channel, it is beneficial to define the maximum permissible quantity (referred to as parameter 11) of commercial-to-spot placements that will advance to subsequent stages. Any excess commercial-to-spot placements are excluded from further consideration. It is important to emphasize that the sorting order varies depending on the heuristic variant employed, highlighting its importance in ensuring consistency and effectiveness throughout the optimization process. Possible sorting criteria for parameter 10 include:

- Rule 11.1: sort the commercial to spot placements in not ascending order of viewership, next, sort the parts inside sorted ones in not ascending order of viewership ratio.
- Rule 11.2: sort the commercial to spot placements in not ascending order of viewership ratio, next, sort the parts inside sorted ones in not ascending order of viewership.

In this context, the viewership ratio is calculated as the ratio of the total viewership of all commercial to spot placements on all channels divided by the spent advertiser budget.

2) Target mushroom size (height, weight).

Considering the average viewership associated with commercial-to-spot placement on a channel, it is essential to establish the minimum viewership threshold for each spent budget value (referred to as parameter 12). Additionally, taking into account the cumulative average viewership of all channels forming the commercial to spot placement on the designated channel, it is necessary to define the minimum total viewership threshold for the future solution (referred to as parameter 13).

3) Picker's basket size.

The size of the solution space, akin to a picker's basket, represents the total number of solutions feasible to explore within the allotted timeframe or the specified time frame designated for solution generation, contingent upon available technical resources. This encompasses considerations such as computational capacity, algorithmic efficiency, and the duration allocated for solution generation.

Find the biggest mushroom among the picked mushrooms in the basket.

At this point, a set of solutions has been generated. The subsequent phase entails identifying and opting for the solution with the highest total viewership among the available set of solutions. This step guarantees the selection of the solution with the utmost viewership, representing the best outcome from the range of generated solutions.

Additional notes for parameter usage.

The differentiation between budget parameters 1 and 2, and parameters 7 and 8, lies in their application within the outlined steps. Parameters 1 and 2 consider the total budget that the advertiser agrees to spend for the advertising campaign, whereas parameters 7 and 8 are derived from the average cost of commercials to spot placement on the channel.

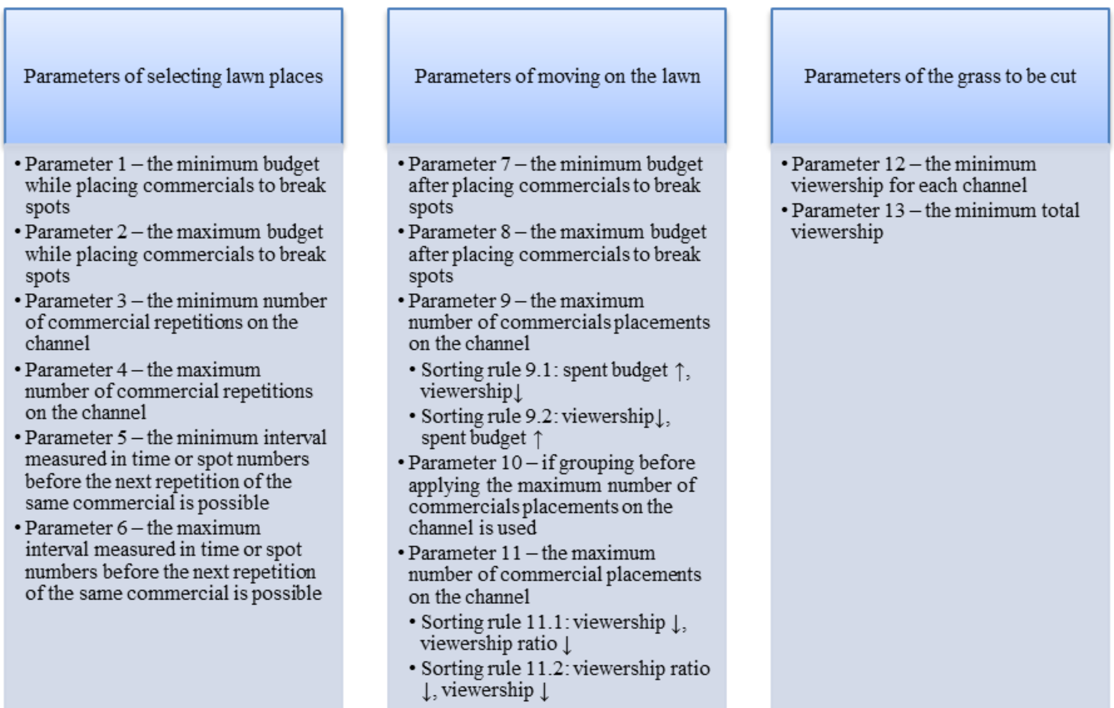
Parameters 9 and 11 have distinct definitions: parameter 9 specifies the number of commercials to spot placement on one channel, permitting variability across all channels, while parameter 11 denotes the total number of commercials to spot placement across all channels, allowing for different values across each channel.

Distinct values may be assigned to different channels for parameters 1-12, reflecting the unique characteristics and requirements of each channel. Parameter 13 represents the total viewership of all break spots across all channels.

By incorporating these parameters, we streamline the optimization process, focusing exclusively on commercials to spot placement that gives the advantage to solutions with higher viewership. Additionally, these parameters serve as a mechanism for quality control, enabling the evaluation and monitoring of algorithmic outcomes. Through systematic assessment and adjustment based on these parameters, we can refine the algorithm's performance over time.

Figure 4 summarizes the explained parameters.

FIGURE 4. MUSHROOM PICKING PARAMETERS IN MEDIA PLANNING PROBLEM



C. Additional notes for instances solution space exploration.

The application of parameter varieties depends on the size of the problem instance under consideration. For smaller instances, in the best cases we can process without any parameter, so all solutions can be generated. This yields an optimal solution. But this could happen very rarely. However, in practice, the instances are large; therefore, a different approach must be used to manage the computational load effectively. As a result, a set of parameters that reduce the solution space must be employed.

The parameters help to find the areas of the whole solution space where profitable solutions are likely to be found. These areas correspond to regions of the metaphorical language "clearings where mushrooms are bigger", signifying their potential to provide high-quality solutions. By identifying and focusing on these areas, we can prioritize our exploration efforts, maximizing efficiency within available computational resources.

So, in summary, depending on the instance size, we adjust our approach to solution exploration. For the smallest instances, we can afford to consider all possible solutions. For medium instances not all possible solutions, but a set of profitable ones could be explored. However, for larger instances, we must adopt a more selective approach, targeting specific regions of the solution space where promising solutions are most likely to be found. This strategy allows us to navigate the complexity of larger instances effectively, ensuring that our computational resources will cope with the task.

VI. Conclusion

Resource allocation presents a widespread challenge in numerous real-world contexts, representing a cornerstone issue within the fields of engineering and operations research. In the context of manufacturing, resources encompass a broad spectrum of elements integral to the production process, spanning materials, tools, machinery, labour, and financial capital. These components collectively contribute to the manufacturing process, ultimately yielding the desired goods or services.

An illustration of how items should be arranged on store shelves or in-store displays to increase sales and improve the shopping experience is called a planogram. It functions effectively as a design for setting up goods in a physical area, accounting for elements like product assortment, positioning, spacing, and marketing collateral. The complexity of planograms can range from rudimentary sketches to intricate digital renderings tailored to the specific requirements and capabilities of the retailer.

Retailers frequently utilize planograms to make sure that their shelves are arranged in a way that draws people in, promotes important items, and makes it simple for customers to discover what they're searching for. Depending on the retailer's objectives and available resources, they may be straightforward sketches or intricate computer designs.

Media planning entails strategically distributing resources to enhance the efficiency of commercial endeavours across diverse channels. In the realm of television media planning, optimization techniques play a critical role in crafting optimal advertising strategies. By leveraging data analytics, predictive modelling, and algorithmic optimization, media planners can identify the most effective combination of commercial placements, scheduling, and budget allocation to maximize the impact of their campaigns.

We propose a novel mushroom-picking heuristics that could be efficiently applied to resource allocation problems, which can be modelled as a knapsack problem with categorized items. We exemplify the application of the proposed mushroom-picking heuristics in addressing the following specific problems. Both of these problems can be conceptualized and modelled using the framework of a knapsack problem:

- Shelf space allocation problem: This pertains to the arrangement of products on shelves within a retail store, where the objective is to specify the quantity of each product on each shelf to maximize the total profit from their sales.
- Media planning problem: This involves the scheduling of TV commercials across various channels during TV spots within breaks between programs, aiming to maximize the total viewership.

Our approach represents a systematic methodological advancement in resource allocation optimization, proposing a structured framework for solution generation. Each mushroom represents one of the possible solutions. Ultimately, our primary objective is to identify the biggest mushroom in the forest. By systematically iterating through the process of identifying mushroom-picking clearings and adjusting mushroom-picking parameters, we aim to optimize our approach to achieving this goal efficiently and effectively.

Imagine the mushroom picker's basket as a metaphorical container holding all the potential solutions to a given problem. Its size is determined by the limitations imposed by technical resources, such as processing speed and memory capacity. Essentially, it represents the boundary within which the exploration of solutions must occur, balancing the desire for thorough exploration with the practical constraints of resource availability. Moreover, the picker watches the basket each time it operates. So, he does not place a new mushroom inside if its quality is worse than the mushrooms already in the basket. The picker's basket denotes the breadth of solutions that can be feasibly explored within a predetermined timeframe, guided by the constraints of available technical resources.

Heuristics are based on the following stages:

- preparing the forest ground;
- identifying mushroom clearings;
- finding the mushrooms;
- deciding if the mushrooms are picket and put into the basket.

By leveraging specific parameters and sorting rules, we aim to guide the optimization process towards identifying solutions that strike a balance between effectiveness and efficiency. This targeted approach not only enhances the robustness of our optimization method but also contributes to reducing computational overhead, making it a viable solution for real-world application scenarios.

We propose 12 tuning parameters for shelf space allocation heuristics, which allow for solution space reduction and for generating high-profitable solutions. They are:

- the minimum and maximum category width while forming allocations (parameters 1, 2);
- the minimum and maximum numbers of items while forming allocations (parameters 3, 4);
- the set of profitable groups of items (parameter 5);
- the minimum and maximum category width after forming allocations (parameters 6, 7);
- the maximum number of allocations with two variants of sorting rule applied (parameter 8);
- grouping option before applying the maximum number of allocations (parameter 9);
- the maximum number of allocations of the category h two

variants of sorting rule applied (parameter 10);
- the minimum profit for each category (parameter 11);
- the minimum total profit (parameter 12).

We propose 13 tuning parameters for media planning heuristics, which help in solution space reduction and generating high-quality solutions. They are:

- the minimum and maximum budget while placing the commercials into the break spots (parameters 1, 2);
- the minimum and maximum number of commercial repetitions on the channel (parameters 3, 4);
- the minimum and maximum interval measured in time or spot numbers before the next repetition of the same commercial is possible (parameters 5, 6);
- the minimum and maximum budget after placing commercials to break spots (parameters 7, 8);
- the maximum number of commercials placements on the channel with two variants of sorting rule applied (parameter 9);
- grouping option before applying the maximum number of commercials placements on the channel is used (parameter 10);
- the maximum number of commercial placements on the channel with two variants of sorting rule applied (parameter 11);
- the minimum viewership for each channel (parameter 12);
- the minimum total viewership (parameter 13).

In addition, no random elements were generated. Establishing these parameters is akin to setting the coordinates on a map, providing clear directions for the algorithm to follow as it navigates through the solution space. Only meaningful parts of solutions or deeply thought-out solutions are created. Through meticulous delineation of these guiding principles, we can guarantee that the algorithm operates with precision, directing its endeavours towards producing superior solutions while evading potential pitfalls.

In essence, by establishing and fine-tuning the steering parameters, we can give way to the complete capabilities of the mushroom-picking algorithm, allowing it to function with accuracy and effectiveness in the pursuit of optimal shelf and product allocations in shelf space allocation problem or spot allocation in media planning problem.

By employing strategic planning, innovative methodologies, and efficient management practices, organizations can navigate resource allocation problems effectively, thereby driving productivity, enhancing performance, and fostering sustainable growth. Our proposed heuristics leverage this inspiration to develop an algorithm that effectively navigates the resource allocation landscape. Our approach aims to enhance the efficiency and accuracy of resource allocation decisions, enabling better adaptation to diverse and dynamic scenarios commonly encountered in real-world applications. Through empirical evaluation and comparative analysis, we demonstrate the effectiveness of our mushroom picker heuristics in solving knapsack-like resource allocation problems with product categorization, showcasing their potential for practical implementation in various domains, including TV media planning and retail shelf space allocation.

Future research can direct the application of the mushroom picking heuristics to optimization in other practical sectors like logistics, inventory management, and supply chain management. Here, based on the nature of the problem, another tuning parameter should be developed.

VII. References

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