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## **APPLICATION OF BLUME METHOD IN FORECASTING RISK ON THE EXAMPLE OF PUBLIC COMPANIES LISTED ON WIG20**

### **Summary:**

*The paper presents the results of studies on the use of Blume's beta to identify systematic risk of companies listed on the Warsaw Stock Exchange. For this purpose, beta values for WIG20 companies for 2014-2016 were calculated. Weekly rates of return on stocks of certain companies were used in the calculations. Once the annual betas were estimated, the author conducted regression of the results to develop an equation that would enable an estimation of parameters for the future period. In most of the analyzed cases, values of beta parameters calculated on the basis of historical data and the data obtained by Blume's method were similar. Therefore, Blume's adjustment method is a good tool for forecasting market risk level of shares of companies listed on the Polish stock exchange.*

**Key words:** shares, systematic risk, beta, Blume's adjustment method

### **Introduction**

Risk is one of the fundamental criteria considered by investors in decision making processes. Therefore, for many years it has been prompting the search for even more efficient methods of its measurement. Numerous suggested solutions include the beta adjustment method developed by Marshall Blume. The application of this method to measure systematic risk of stock exchange shares is presented on the example of WIG20 companies listed on the Warsaw Stock Exchange. These companies were selected because their shares are the most traded securities on the Warsaw Stock Exchange. For this purpose, beta values for the companies were estimated and used to measure the systematic risk of shares. The factors were then used to create a regression equation,

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according to the assumptions of the beta adjustment method developed by Marshall Blume<sup>1</sup>.

Beta values for the respective WIG20 companies, calculated by means of linear regression and comparison to historical data, are presented in the paper. The aim of the article is to present the use of Blume's beta adjustment method in predicting investment risk and creating an investment portfolio. The author intends to show predictions of betas for a subsequent period. This will allow to predict the risk of investing in each of the analyzed companies.

### **1. Beta factor — measure of non-diversifiable market risk**

Risk is an integral element of every investment. Investors give up their present, sure consumption for a future but uncertain one. The category of risk has many definitions proposed by representatives of different fields of science. In finances, risk is particularly associated not only with a possible loss but also with an opportunity to achieve results other (better) than expected<sup>2</sup>.

The most commonly used measure used to assess an investment is the rate of return. In case of investing in company shares, risk is defined as potential variability of the return rates on investment. Accordingly, share portfolio risk should be defined as the total variability of return rates on portfolio elements. Thus, it will consist of non-diversifiable (systematic) market risk and diversifiable specific risk<sup>3</sup>.

Specific risk (also called diversifiable or eliminable) is the risk typical only for a certain security. Systematic risk (market, non-diversifiable) is impossible to eliminate. It is caused by factors that affect the prices of all other traded shares<sup>4</sup>.

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<sup>1</sup> Blume M., *On the assesment of risk*, The Journal of Finance, Vol. XXVI, No 1 1971, p.1-10.

Blume M., *Betas and their regression tendencies*, The Journal of Finance, Vol. XXX, No 3 1975, p.785-795.

<sup>2</sup> Jajuga K., Jajuga T., *Inwestycje*, PWN, Warszawa 2004, p. 99-100.

<sup>3</sup> Feder-Sempach E. (2012), *Analiza porównawcza prognozy dla wybranych spółek z indeksu WIG20* [in:] J. Handschke, W. Przybylska-Kapuścińska (ed.), *Rynki finansowe i ubezpieczenia: Nowe perspektywy instytucji i instrumentów*, the University of Economics in Poznań, no 143/2010 p. 82-83.

<sup>4</sup> Feder-Sempach E. (2017) *Efekt interwału w oszacowaniach współczynnika beta na podstawie akcji spółek z indeksu WIG20 I DAX w okresie 2005-2015-Analiza*

This part of comprehensive risk indicates that there is a common factor affecting the prices of all shares. William Sharpe proposed a single-index model that assumes that there is only one factor influencing the variability of return rates on shares<sup>5</sup>. It is assumed that this factor was the market index (in this paper it is the WIG broad market index). Sharpe also assumed that the relationship between the changes in the rates of return on a security and changes in the rates of return of the said index is linear.

According to simplifications suggested by Sharpe, relations between changes in a company's rate of return and changes in this market rate can be calculated by comparing the first value to the rate of return on the applied stock exchange index. Sharpe developed a model equation which, after being rearranged, grouped the systematic (market) and specific risk factors for a given rate of return. The equation is presented below. It determines a security characteristic line (SCL) and indexes the  $R_i$  level.

$$R_i = \beta_i R_m + \alpha_i + e_i$$

where:

$\beta_i, \alpha_i$  — model parameters,

$R_m$  — market rate of return identified with the WIG index rate of return,

$e_i$  — random element with expected value of 0.

The first part of the equation defined by the  $\beta_i R_m$  product symbolizes the non-diversifiable part of return rate variability. It means a certain, fixed level of market risk that cannot be eliminated. The sum  $\alpha + e_i$  is the diversifiable part of the return rate variability<sup>6</sup>. According to the assumptions of portfolio theory by Markowitz, in a well diversified portfolio it can be totally eliminated<sup>7</sup>.

The subject of this paper is the assessment of beta, therefore the author focuses on the first part of the equation.

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*porównawcza*, Economic Studies. Scientific Journal of the University of Economics in Katowice, no 325/2017, p. 20-23.

<sup>5</sup> Sharpe W., *A Simplified Model for Portfolio Analysis*, Management Science, Vol. IX, No 2 1963, pp. 277-293.

<sup>6</sup> Węgrzyn H. (2016), *Szacowanie kosztu kapitału własnego przedsiębiorstwa z branży telekomunikacyjnej z wykorzystaniem metody AHP*, WSH Scientific Journal. Management, No 3/2016, pp. 404-405.

<sup>7</sup> Markowitz H., *Portfolio selection. Efficient diversification of investment*, Cowles Foundation for Research in Economics at Yale University, New York 1959, pp. 72-102.

The beta factor (called the aggressiveness factor) is a measure of risk associated with investing in company shares. Its level depends, among others, on the structure of assets, nature of the business or available financing sources<sup>8</sup>. It shows the average change (in percentage points) of the rate of return that will occur as a result of an increase of the market return rate by one percentage point. This parameter serves the assessment of the degree of market risk of various assets<sup>9</sup>. Depending on the values of the  $\beta$  parameter, there are:

- aggressive shares with  $\beta > 1$  — the change in the company's rate of return is higher than the change in the market rate of return,
- defensive shares with  $\beta < 1$  — the change in the company's rate of return is lower than the change in the market rate of return,
- shares with  $\beta < 0$  — the change in the company's rate of return moves in the opposite direction than the change in the market rate of return.

There are also assets with  $\beta = 1$  and  $\beta = 0$ . As for the first, the change in the rate of return is identical with the change in the market rate of return. In case of the other, the change in the market rate of return does not imply changes in the company's rate of return<sup>10</sup>.

The beta parameter reflects many market conditions analyzed by stock investors. These conditions translate directly into the demand for shares and thus, their market valuation<sup>11</sup>.

Investors who want to apply Sharpe's single-index model should estimate betas for each share that is supposed to be in their portfolio. As mentioned, this is one of the most important measures of market risk. The  $\beta$  coefficient is most often determined on the basis of historical data. Based on it, the beta value for future periods is forecasted. It has been proved that past data help to estimate future factor values. There is a method of adjusting betas calculated by means of a historical method,

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<sup>8</sup> Rydzewska A. (2016), *Contemporary nature of stock exchange from the prospective of demutualization process*, *Oeconomia Copernicana*, Vol. 7, issue 1, 2016, pp. 49-51.

<sup>9</sup> Brealey R.A., Myers S.C., *Podstawy finansów przedsiębiorstw* [original title: *Principles of Corporate Finance*], Vol. I and II, translated from English by Jajuga K., Jajuga T., *Inwestycje*, PWN, Warszawa 1999, pp. 99-100.

<sup>10</sup> Pera K., Buła R., Mitrenga D., *Modele inwestycyjne*, C. H. Beck, Warszawa 2014, p. 130.

<sup>11</sup> Jonek-Kowalska I. (2017), *Pomiar i ocena ryzyka w przedsiębiorstwach górniczych-perspektywa międzynarodowa*, *Scientific Journal of the Silesian Technical University*, No 1974/2017, pp. 212-214.

that takes into account their movement towards the average.<sup>12</sup> This method was developed by Marshall Blume.

## 2. Marshall Blume's beta adjustment

Estimating the values of  $\beta$  parameters on the basis of historical data involved an error of estimation. The received estimates often led to significant discrepancies and that did not allow to obtain reliable assessments.

When  $\beta$  factors are studied, the assessment of the relationship between them and the two adjacent periods is an important issue. One of the methods that enabled more precise parameter estimation was proposed by Marshall Blume. He studied portfolios consisting of no more than one hundred shares listed on the New York Stock Exchange in the years 1926-1968 (6 study periods, seven years each) and estimated the betas for each portfolio. Then, he conducted a linear regression of the estimated parameters. The independent variable was  $\beta$  calculated for a preceding period, whereas the dependent variable<sup>13</sup> was  $\beta$  for a following period. His studies implied that values of forecasted betas calculated by means of regression were much closer to one than the values obtained from historical data. This may lead to a conclusion that  $\beta$  parameters for certain companies actually move much closer to the value of  $\beta=1$ , identified with the broad market beta.

According to Blume's research, the level of the  $\beta$  factor may be estimated using the following equation<sup>14</sup>.

$$\beta_{iu+1}^P = \gamma + \varphi \beta_{iu}$$

Where:

$\beta_{iu+1}^P$  — predicted beta value in the period following the analyzed one,

$\beta_{iu}$  — historical beta value for the analyzed period,

$\gamma, \varphi$  — model parameters obtained by means of linear regression.

Once  $\gamma$  and  $\varphi$  parameters are calculated by means of regression, investors can estimate the value of beta in the next period by replacing

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<sup>12</sup> Blume M., *On the... Op. cit.*, pp. 1-10.

<sup>13</sup> Welfe A., *Ekonometria*, PWE, Warszawa 2003, pp. 23-39.

<sup>14</sup> Pera K., Buła R., Mitrenga D., *Op. cit.*, p. 144.

the parameter value recorded in the analyzed period. This procedure was adopted to estimate the  $\beta$  factors for blue chips listed on the Warsaw Stock Exchange in the empirical section of this paper. Calculation results and forecasts of parameters for the next period are presented therein.

Before the study results are presented, it is worth to mention another  $\beta$  adjustment method. It was developed by Oldrich Vasicek<sup>15</sup>. He took into account the variation of average errors of estimation that occur during  $\beta$  estimation. He claimed that if there is an increased uncertainty regarding parameters estimations, the scale of adjustment must also be bigger. However, this paper's objective is to present the use of Blume's adjustment method, therefore Vasicek's adjustment method will not be further discussed.

### **3. Systematic risk assessment for WIG20 companies**

In this section,  $\beta$  coefficients for companies listed in the WIG20 index will be presented. The companies and the industry sectors they belong to are listed in the table below<sup>16</sup>.

**Table 1. WIG20 companies and their industry sectoral classification — WIG20 index as of 26.12.2016.**

<b>Company name</b>	<b>Industry macrosector</b>	<b>Industry sector</b>
ALIOR BANK	FINANCES	Banks
ASSECO	TECHNOLOGIES	IT
BZWBK	FINANCES	Banks
CCC	CONSUMPTION GOODS	Clothing and cosmetics
CYFROWY POLSAT	TECHNOLOGIES	Telecommunications
ENEA	OIL AND ENERGY	Energy
ENERGA	OIL AND ENERGY	Energy
EUROCASH	TRADE AND SERVICES	Wholesale trade
KGHM	CHEMISTRY AND RAW MATERIALS	Mining

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<sup>15</sup> Vasicek O., *A note on using cross-sectional information in Bayesian estimation of security betas*, The Journal of Finance, Vol. 28, No 5 1973, pp. 1233-1239.

<sup>16</sup>According to the new sectoral classification of the Warsaw Stock Exchange, established on 15.12.2016.

Company name	Industry macrosector	Industry sector
LOTOS	OIL AND ENERGY	Oil and gas
LPP	CONSUMPTION GOODS	Clothing and cosmetics
MBANK	FINANCES	Banks
ORANGE	TECHNOLOGIES	Telecommunications
PEKAO	FINANCES	Banks
PGE	OIL AND ENERGY	Energy
PGNIG	OIL AND ENERGY	Oil and gas
PKN ORLEN	OIL AND ENERGY	Oil and gas
PKOBP	FINANCES	Banks
PZU	FINANCES	Other
TAURON	OIL AND ENERGY	Energy

Source: Author's own compilation based on [https://www.gpw.pl/akcje\\_i\\_pda\\_notowania\\_ciagle](https://www.gpw.pl/akcje_i_pda_notowania_ciagle) (access on: 26.12.2016).

Betas for the companies listed in Table 1 were calculated based on the share closing prices. Weekly rates of return were used in the calculations.

In order to verify the assumptions of Blume's  $\beta$  adjustment, the analyzed period 2014-2016 was divided into three identical study periods, 52 weeks each.

Betas calculated for each of the subperiods are presented in Table 2. To calculate the factors and the regression equation, Microsoft Excel functions were applied<sup>17</sup>.

**Figure 2. Calculated  $\beta$  values in certain subperiods.**

Company name	$\beta$ in 2014 subperiod I	$\beta$ in 2015 subperiod II	$\beta$ in 2016 subperiod III
ALIOR	1.14	1.10	2.03
ASSECO	0.65	0.35	0.40
BZWBK	1.21	1.50	1.21
CCC	1.37	1.16	1.16

<sup>17</sup> Anholcer M., Gaspars-Wieloch H., Owczarkowski A., *Ekonometria z Excelem. Przykłady i zadania*, the University of Economics in Poznań, Poznań 2012, pp. 145-179.

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<b>Company name</b>	<b><math>\beta</math> in 2014 subperiod I</b>	<b><math>\beta</math> in 2015 subperiod II</b>	<b><math>\beta</math> in 2016 subperiod III</b>
CYFROWY POLSAT	0.69	1.07	0.64
ENEA	1.14	0.97	1.18
ENERGA	1.06	1.02	1.39
EUROCASH	0.53	0.73	0.91
KGHM	1.01	2.20	2.21
LOTOS	2.06	1.59	1.15
LPP	0.41	1.13	0.65
MBANK	1.49	1.48	1.15
ORANGE	0.34	1.19	0.65
PEKAO	0.95	1.00	1.23
PGE	0.96	0.94	1.00
PGNIG	1.30	1.12	1.01
PKN ORLEN	0.87	0.75	1.23
PKOBP	1.14	1.35	1.37
PZU	0.84	1.22	1.28
TAURON	0.98	1.03	0.98

*Source: Author's own compilation based on archive quotations at [www.stooq.pl](http://www.stooq.pl) (access on 22.12.2016).*

Based on the previously mentioned company classification due to their beta level, the value of the coefficient was below 1 in all analyzed periods for only 3 companies (ASSECO, EUROKASH, PGE). This implied that their actual change in the rate of return was lower than the change in the WIG rate of return. Thus, they can be classified as defensive companies.

For as many as 9 companies (ALIOR, BZWBK, CCC, ENERGA, KGHM, LOTOS, MBANK, PGNIG, PKOBP) the beta was more than 1, which suggested that that changes in the rates of return were bigger than the changes in the broad market indices. These companies can be classified as aggressive. In case of the remaining 8 companies, the parameter values in the analyzed periods varied.

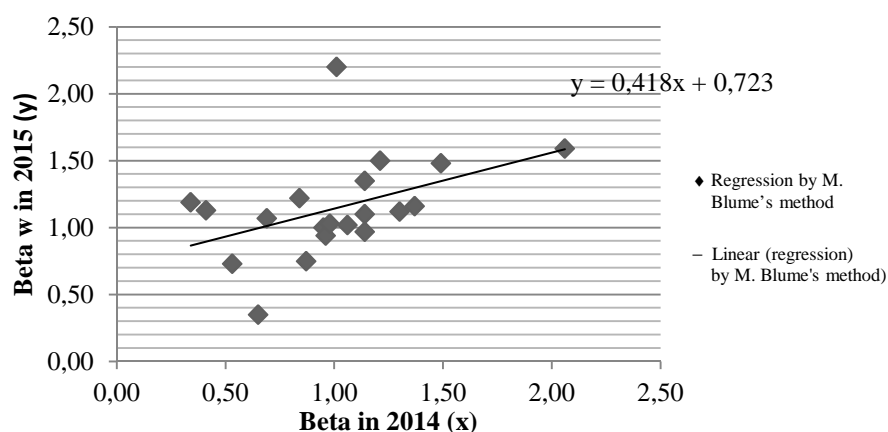
Once the  $\beta$  parameters for the WIG20 companies for 2014, 2015 and 2016 were estimated, linear regression was conducted. The independent variable was the beta for the preceding period (for example, 2014) and



the dependent variable was the beta for the following period (2015). A procedure similar to the one used by Marshall Blume in his studies was applied.

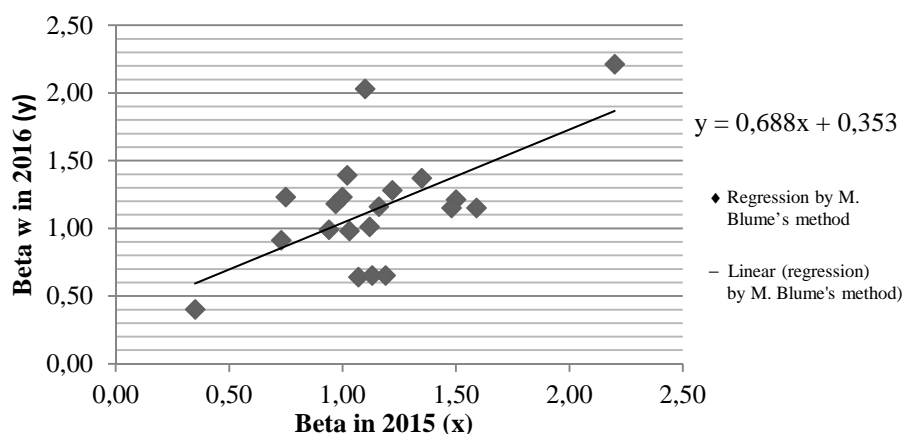
The analysis of 3 periods allowed to estimate two regression equations. The first one was developed based on the parameters from 2014 and 2015 and the other one was formulated by using coefficients estimated for 2015 and 2016.

**Figure 3. Linear regression of the  $\beta$  coefficient from the first sub-period (year 2014) compared to the second research sub-period (2015).**



Source: Own study.

**Figure 4. Linear regression of the  $\beta$  coefficient from the 2nd sub-period (year 2015) compared to the third research sub-period (2016)**



Source: Own study.

Figures 3 and 4 presented here show the results of calculations of equations. In the first of them there is a regression equation calculated using the  $\beta$  parameters from 2014 and 2015, while in the second one, as mentioned earlier, the equation calculated using these coefficients from 2015 and 2016.

The first regression equation was accepted in the form:

$$\beta_{2016}=0,418\beta_{2015}+ 0,723$$

where:

$\beta_{2016}$ - forecast of the beta parameter value in 2016

$\beta_{2015}$ - the historical value of the beta parameter in 2015.

In order to approximate the operation of the Blume correction for the shares of the WIG20 index companies, you can use an example. Beta i-company recorded in 2015 it was 2 (hypothetical value). After inserting this value into the above equation, the forecasted beta for this company for 2016 will amount to 1.559. In another beta example of the j-company in 2015, it was 0.5. The estimated value of the parameter for the following year was 0.932. What Blume has already noted in his research has the tendency to move towards unity, and thus the value of the market beta. Its correction leads to lowering the high values of the indicator and similarly raising the low<sup>18</sup>.

According to the presented example, the forecasts of  $\beta$  index indicators of the WIG20 index for 2016 were calculated. Their values in the confrontation with the coefficients calculated on the basis of the weekly return rates, see Figure 5.

**Figure 5. Forecast of  $\beta$  parameters for 2016 (calculated using the Blume method) together with real  $\beta$  coefficients in 2015-2016**

Company name	$\beta$ in 2015 Subperiod II	Forecast of $\beta$ Blume's method for 2016	$\beta$ in 2016 Subperiod III
ALIOR	1,10	1,18	2,03
ASSECO	0,35	0,87	0,40
BZWBK	1,50	1,35	1,21

<sup>18</sup> Feder- Sempach E., Op. cit., p. 88.

Company name	$\beta$ in 2015 Subperiod II	Forecast of $\beta$ Blume's method for 2016	$\beta$ in 2016 Subperiod III
CCC	1,16	1,21	1,16
CYFROWY POLSAT	1,07	1,17	0,64
ENEA	0,97	1,13	1,18
ENERGA	1,02	1,15	1,39
EUROCASH	0,73	1,03	0,91
KGHM	2,20	1,64	2,21
LOTOS	1,59	1,39	1,15
LPP	1,13	1,20	0,65
MBANK	1,48	1,34	1,15
ORANGE	1,19	1,22	0,65
PEKAO	1,00	1,14	1,23
PGE	0,94	1,12	1
PGNIG	1,12	1,19	1,01
PKN ORLEN	0,75	1,04	1,23
PKOBP	1,35	1,29	1,37
PZU	1,22	1,23	1,28
TAURON	1,03	1,15	0,98

Source: Own study

Analyzing the above data in the majority of cases, one can notice a tendency to strive for unity of the beta parameters estimated using the Blume method in relation to their calculations from historical data. What also seems relevant in no case was the negative value of the parameter. This proves that the rates of return of individual WIG20 companies have been followed up with the broad market index.

The assignment of companies to the aggressive ( $\beta > 1$ ) and defensive categories ( $\beta < 1$ ) described above was confirmed in the position relative to unity in the estimation of beta parameters by the Blume method. Only 5 companies (CYFROWY POLSAT, EUROCASH, LPP, ORANGE, TAURON) changed their position in relation to the neutral beta of one in the calculation of coefficients with both methods. Therefore, they cannot be unequivocally qualified as aggressive or defensive companies.

The highest compliance of the level of risk measured by two methods of calculating the beta coefficient can be seen in the case of CCC, ENEA and PZU securities. In all cases, the forecast value from estimates using historical data differed no more than by 0.05. The smallest correspondence, however, was found in the case of ALIORA papers.

As presented in the previous drawings, the Blume method was used for two regression equations. The second one is as follows:

$$\beta_{2017} = 0.688\beta_{2016} + 0.353$$

where:

$\beta_{2017}$  - forecast of the beta parameter value in 2017,

$\beta_{2016}$  - the historical value of the beta parameter in 2016

This equation will serve to predict  $\beta$  parameters for 2017. Their estimates are presented in Figure 6.

Looking at the beta parameter forecasts for 2017, a risky investor can assess which companies can bring a higher return on the broad market index in the event of a boom in the stock market (in the case of a bear market, the rate of return will be lower). These are those whose prediction of  $\beta$  parameters has exceeded the value of one. On the other hand, an investor with a risk aversion will choose companies with lower beta (ASSECO, CYFROWY POLSAT, EUROCASCH, LPP, ORANGE).

**Figure 6. Forecast of  $\beta$  parameters for 2017 (calculated using Blume's method) together with real  $\beta$  coefficients in 2016.**

<b>Company name</b>	<b><math>\beta</math> in 2016 subperiod III</b>	<b>Forecast <math>\beta</math> with Blume's method for 2017</b>
ALIOR	2,03	1,17
ASSECO	0,40	0,63
BZWBK	1,21	1,19
CCC	1,16	1,15
CYFROWY POLSAT	0,64	0,79
ENEA	1,18	1,17
ENERGA	1,39	1,31
EUROCASH	0,91	0,98
KGHM	2,21	1,87

Company name	$\beta$ in 2016 subperiod III	Forecast $\beta$ with Blume's method for 2017
LOTOS	1,15	1,14
LPP	0,65	0,80
MBANK	1,15	1,14
ORANGE	0,65	0,80
PEKAO	1,23	1,20
PGE	0,99	1,03
PGNIG	1,01	1,05
PKN ORLEN	1,23	1,20
PKOBP	1,37	1,30
PZU	1,28	1,23
TAURON	0,98	1,03

Source: Own study.

## Conclusions

The author's intention was to show the possibility of measuring the systematic risk of the shares of the largest Warsaw Stock Exchange companies grouped in the WIG20 index using beta coefficients. The weekly historical return rates were used for this purpose and using them, the  $\beta$  parameters for the appropriate test periods were calculated. Then, the results were regressed in accordance with the Blume correction assumptions. On the basis of the results presented in the empirical part, it can be concluded that the applied method in a large part of the surveyed companies showed results comparable to those that took place in fact. Despite some differences between the forecasts of the  $\beta$  parameter and their estimates from historical data, in the vast majority of cases, the company could be classified as aggressive or defensive. This information gives clear suggestions to particular types (due to the attitude to risk) of investors which shares should be in their portfolio.

Summing up, the presented calculation procedure created by Marshall Blume can serve as a tool allowing for relatively simple risk measurement on the stock market.

### **Legal acts**

- [1.] Resolution of 15/12/2016 No. 1343/2016 of the Management Board of the Warsaw Stock Exchange regarding the sectoral classification of listed companies and companies whose financial instruments are introduced to the alternative trading system.

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